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A PROBABILITY MODEL FOR INITIAL CRACK SIZE AND FATIGUE LIFE OF --ETC(U)  
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The Florida State University

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**ABSTRACT**

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With constant firing, metal fatigue produces cracks in a gun barrel. The useful life of the barrel comes to an end when a crack develops to a critical size. The theory of Fracture Mechanics suggests a formula for crack size growth rate. This formula can be used to determine the life of a barrel depending on the initial and critical crack sizes and other factors. The initial crack size turns out to be a dominant factor. Unfortunately, accurate measurements are not generally available on the initial crack size. In this paper, <sup>is proposed</sup> we propose a simple probability model for the initial crack size and this, in turn, leads to a probability distribution of the life of the barrel. This last distribution is the well known exponential distribution with a location shift. The simplicity of this final result is one of the factors that make the model appealing.

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# A PROBABILITY MODEL FOR INITIAL CRACK SIZE AND FATIGUE LIFE OF GUN BARRELS

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F. Proschan and J. Sethuraman  
The Florida State University

1. Introduction. Suppose that there is a crack of size  $b$  in a gun barrel after  $N$  rounds have been fired. The rate at which the crack size will grow,  $\frac{db}{dN}$ , is a central topic of study in Fracture Mechanics. The following formula, taken from Davidson, Throop and Reiner (1972), represents a reasonably simplified version of many more complicated ones available.

$$(1.1) \quad \frac{db}{dN} = \frac{C(\Delta K)^m}{E S_Y K_{Ic}}$$

where

$\Delta K$  = increase in stress intensity  
 $E$  = elastic modulus  
 $S_Y$  = tensile yield strength  
 $K_{Ic}$  = fracture toughness  
 $m$  = a constant between 2 and 4  
 $C$  = a material constant.

Throop (1972) gives a simple expression for  $\Delta K$ , which depends on the size  $b$  and the shape of the crack, as follows:

$$(1.2) \quad \Delta K = \alpha S \sqrt{\pi b},$$

where

$S$  = bore stress =  $p \frac{\omega^2 + 1}{\omega^2 - 1}$ ,  
 $p$  = pressure  
 $\omega$  = ratio of outer diameter to inner diameter  
 $\alpha = \begin{cases} 1.5 & \text{if the crack is a frontal notch} \\ 1.0 & \text{if the crack is semi-elliptical} \\ 0.5 & \text{if the crack is semi-circular.} \end{cases}$



Thus

$$(1.3) \quad \frac{db}{dN} = D b^{m/2},$$

where

$$(1.4) \quad D = [C \alpha^m S^m \pi^{m/2}] / [E S_y K_{Ic}].$$

Suppose that the barrel is fired a few rounds to provide a heat check pattern. The size of the largest crack produced at this stage is called the initial crack size and denoted by  $b_0$ . The number of rounds,  $L$ , required for this crack to grow to the critical crack size  $b_c$  is the life of the barrel. From (1.3), it is easy to obtain an expression for  $L$ :

$$(1.5) \quad L = \begin{cases} \frac{2}{(m-2) D} [b_0^{-(m-2)/2} - b_c^{-(m-2)/2}] & \text{if } m > 2. \\ \frac{1}{D} [\log b_c - \log b_0] & \text{if } m = 2. \end{cases}$$

It is easy to see from (1.5) that while the effect of  $b_c$  on  $L$  is minimal, the effect of  $b_0$  is significant. For instance, an increase from 4 in. to 5 in. in  $b_c$  increases  $L$  only by  $\frac{1}{200}$ , when  $m = 4$ . For the same  $m$ , the increase in  $L$  when  $b_0$  is decreased from .01 in. to .001 in., is  $\frac{900}{D}$ . Also, the initial crack size is difficult to measure since it is so small. It would therefore be useful to model the distribution of  $b_0$ . We do this in the next section and show how this leads to a simple distribution for  $L$ .

To the best of our knowledge, there have been only two previous attempts made to introduce probability models in the problem of fatigue crack growth. Racicot (1976) assumed fixed values for  $b_0$  and  $b_c$ , and various probability distributions for the several factors in  $D$ . He then obtained empirical results

for the distribution of  $L$  by simulation on a computer. Hanagud and Uppaluri (1976) used a formula for crack growth rate which is slightly more complex than (1.1) and involves a quantity  $r$  which is the ratio of the maximum stress intensity factor to the minimum stress intensity factor. They assumed that both  $\Delta K$  and  $r$  were random, and obtained approximations to the expected crack size growth rate. They used these results to study improvements in reliability obtainable by examination and repair. Since examination and repair are not possible in a gun barrel operating in the field, their results are not applicable to our problem.

## 2. The probability model.

The first few rounds of firing produce a heat check pattern. Let this pattern contain  $N$  cracks having sizes  $C_1, \dots, C_N$ . Then the initial crack size  $b_0$  is given by

$$b_0 = \max \{C_1, \dots, C_N\}.$$

In our probability model, we assume that  $C_1, C_2, \dots$  are independent and identically distributed according to a uniform distribution on  $[0, B]$ . We also assume that  $N$  is an independent variable governed by a Poisson distribution with parameter  $\lambda$ . A physical interpretation for  $B$  and  $\lambda$  is as follows:  $B$  represents the maximum possible initial crack size and  $\lambda$  is a measure of the number of cracks in the heat check pattern. Our probability model depends on just these two easily interpretable quantities.

Thus,

$$\begin{aligned}
 P(b_0 \leq b) &= P(\max(C_1, \dots, C_N) \leq b | N \geq 1) \\
 &= \frac{\sum_{n=1}^{\infty} P(\max(C_1, \dots, C_n) \leq b) P(N = n)}{\sum_{n=1}^{\infty} P(N = n)} \\
 &= \frac{\sum_{n=1}^{\infty} \left(\frac{b}{B}\right)^n e^{-\lambda} \lambda^n / n!}{\sum_{n=1}^{\infty} e^{-\lambda} \lambda^n / n!},
 \end{aligned}$$

so that

(2.1)

$$P(b_0 \leq b) = \frac{e^{\lambda b/B} - 1}{e^{\lambda} - 1}, \quad 0 \leq b \leq B.$$

The distribution in (2.1) is our probability model for the initial crack size. It is physically motivated and simple. A sample application of this model given at the end of this section provides more support for the model.

If the distribution of  $D$  (for definition see (1.4)) or the distribution of the factors that enter  $D$  are known, one can combine that knowledge with the probability distribution of  $b_0$  in (2.1) to obtain the distribution of the life  $L$  of the barrel.

As a sample application, we will obtain an approximation to the distribution of  $L$ . Assume that  $D$  is known and  $m = 4$ . Formula (1.5) states that

$$L = \frac{1}{Db_0} - \frac{1}{Db_c}.$$



Thus, for any  $x > 0$ ,

$$\begin{aligned}
 & P\left\{L \leq \left(\frac{1}{DB} - \frac{1}{Db_c}\right) + \frac{x}{\lambda DB}\right\} \\
 &= P\left\{\frac{b_0}{B} \geq \left(1 + \frac{x}{\lambda}\right)^{-1}\right\} \\
 &\doteq P\left\{\frac{b_0}{B} \geq 1 - \frac{x}{\lambda}\right\} \\
 &= \frac{e^{\lambda} - e^{\lambda(1 - \frac{x}{\lambda})}}{e^{\lambda} - 1} \quad \text{from (2.1)} \\
 (2.2) \quad &\doteq 1 - e^{-x},
 \end{aligned}$$

where  $\doteq$  stands for approximately equal to and we have used the relations

$$\left(1 + \frac{x}{\lambda}\right)^{-1} \doteq 1 - \frac{x}{\lambda} \quad \text{and} \quad e^{\lambda} - 1 \doteq e^{\lambda},$$

which are good approximations even for moderate values of  $\lambda$ . The conclusion in (2.2) may be summarized as follows:

If the number of all crack sizes in the heat check is moderately large, the life of a gun barrel has an exponential distribution with a location shift.

In the general case, where  $\eta$  is assumed to be random, the distribution of the life of the barrel becomes a mixture of shifted exponential distributions.

The assumptions of a uniform distribution for initial crack size and a Poisson distribution for the number of cracks in the heat check pattern are not required for a result like (2.2) and were made here just to illustrate



the proof. In fact, it can be shown from the theory of limiting distributions for the maximum (or minimum) of large numbers of random variables that the distribution of  $L$  can be approximated by a shifted Weibull distribution. In the general case where  $D$  is assumed to be random, the distribution of  $L$  can be approximated by a mixture of shifted Weibull distributions with a common shape parameter.

It is hoped that currently available data on gun barrels will be studied further to test the validity of our model.

### 3. Conclusions.

- (1) A simple probability model for the initial crack size is given in (2.1).
- (2) It is derived, as a consequence, that the life of a barrel has a shifted exponential distribution.
- (3) Under practically no assumptions, it can be shown that the life of a barrel has a shifted Weibull distribution.
- (4) When  $D$  as defined in (1.4) is assumed to be random, the distribution of the life of a barrel is a mixture of shifted Weibull distributions with a common shape parameter.

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20. ABSTRACT

With constant firing, metal fatigue produces cracks in a gun barrel. The useful life of the barrel comes to an end when a crack develops to a critical size. The theory of Fracture Mechanics suggests a formula for crack size growth rate. This formula can be used to determine the life of a barrel depending on the initial and critical crack sizes and other factors. The initial crack size turns out to be a dominant factor. Unfortunately, accurate measurements are not generally available on the initial crack size. In this paper, we propose a simple probability model for the initial crack size and this, in turn, leads to a probability distribution of the life of the barrel. This last distribution is the well known exponential distribution with a location shift. The simplicity of this final result is one of the factors that make the model appealing.